Energetics of rocked inhomogeneous ratchets

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(Received 29 June 2001; published 21 February 2002)

We study the efficiency of frictional thermal ratchets driven by a finite frequency driving force and in contact with a heat bath. The efficiency exhibits varied behavior with driving frequency. Both nonmonotonic and monotonic behavior have been observed. In particular, the magnitude of the efficiency in the finite frequency regime may be more than the efficiency in the adiabatic regime. This is our central result for rocked ratchets. We also show that for the simple potential we have chosen, with only spatial asymmetry (homogeneous system) or only a frictional ratchet (symmetric potential profile), the adiabatic efficiency is always more than in the nonadiabatic case.

DOI: 10.1103/PhysRevE.65.037105 PACS number(s): 05.40.−a, 05.60.−k

Ratchet models (Brownian motors) have been much studied to determine how directed motion appears out of nonequilibrium fluctuations in the absence of any net macroscopic force. Here athermal fluctuation combined with spatial or temporal anisotropy conspire to generate systematic motion even in the absence of net bias [1]. These studies were inspired by observations on molecular motors in biological systems [2]. To this effect several physical models have been proposed under the name of rocking ratchets, flashing ratchets, diffusion ratchets, correlation ratchets, frictional ratchets [1], etc. In most of these systems, the focus was mainly on the behavior of the probability current with change in a system parameter like the temperature, amplitude of the external force, correlation time, etc. The efficiency with which these ratchets convert fluctuation to useful work is a subject of much recent interest [3,4]. New questions regarding the nature of heat engines (reversible or irreversible) at molecular scales are being investigated. In particular, the source of irreversibility and whether the irreversibility can be suppressed such that the efficiency can approach that of the Carnot cycle [2,5] and the generalization of thermodynamical principles to the nonequilibrium steady state are being investigated [6]. We use the method of stochastic energetics developed by Sekimoto [3]. In this scheme, quantities like heat, work done, and input energy can be calculated within the framework of the Langevin equation. Using this approach the efficiency has been studied mainly as a function of temperature and load in rocking, oscillating, and frictional ratchets. In some cases it has been shown that the efficiency can be maximized at a finite temperature [4,7]. The efficiency in these systems is rather small, the reason being the inherent irreversibility of these engines due to finite current.

In our present work we mainly explore the nature of efficiency in frictional rocking ratchets as a function of the frequency of the external drive. A systematic study of efficiency as a function of frequency in rocked ratchets has not been undertaken so far. We show in the following that a rocking ratchet with inhomogeneous friction coefficient can have efficiency that is a nonmonotonic function of frequency. In some parameter range, *the efficiency in the nonadiabatic regime can even be larger than in the adiabatic regime.* This is solely due to the interplay between the asymmetry in the potential and the space dependent friction coefficient. In the absence of frictional inhomogeneity our system reduces to a conventional rocked ratchet. It may also happen that, in spite of this nonmonotonic behavior with frequency, the adiabatic efficiency is larger than the nonadiabatic efficiency. This shows that in the nonequilibrium regime efficiency exhibits complex behavior, some of which is contrary to established tenets of equilibrium phenomena, such as that the efficiency in quasistatic processes is maximum. It is difficult to find any systematic principle or procedure that can optimize the efficiency.

Transport properties in overdamped inhomogeneous systems have been dealt with in great detail previously [8–10]. The occurrence of multiple current reversals [11], current reversal under adiabatic or deterministic conditions, unidirectional motion in the absence of a potential [10], and stochastic resonance in the absence of periodic forcing have also been observed [12]. Most of these phenomena arise solely due to the presence of frictional inhomogeneity. It is to be noted that systems with space dependent friction are not uncommon. Brownian motion in confined geometries shows space dependent friction. Particles diffusing close to a surface have a space dependent friction coefficient [11]. It is also believed that molecular motors move closely along the periodic structure of microtubules and will therefore experience a position dependent mobility [10]. Frictional inhomogeneities are common in superlattice structures and semiconductor systems [8].

We consider an overdamped Brownian particle moving in an inhomogeneous one-deimensional ratchet like a potential, rocked by a finite frequency driving force. We consider an asymmetric potential of the form $V(x) = -1/(2\pi)[\sin(2\pi x) + \mu/4\sin(4\pi x)] + Lx$, where L is the external load against which the Brownian particle moves on average. μ is the asymmetry parameter and is in the range 0–1. The direction of the load is chosen against the mean drift of the Brownian particle so that the work done by the particle is positive. The system is rocked by a zero mean external force of the form $F(t) = A\sin(\omega t)$. The correct Langevin equation for such a

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motion has been derived using a microscopic treatment of the system-bath coupling [9,13]:

$$\dot{x} = -\frac{V'(x) - F(t)}{\eta(x)} - k_B T \frac{\eta'(x)}{[\eta(x)]^2} + \sqrt{\frac{k_B T}{\eta(x)}}\xi(t). \quad (1)$$

The quantity x represents the spatial position of the system. It should be noted that the above equation involves a multiplicative noise with an additional temperature dependent drift term which turns out to be essential for the system to approach the correct thermal equilibrium state in the absence of external drive $F(t)$ and load L [9,13]. The Gaussian white noise $\xi(t)$ is δ correlated with mean zero, i.e., $\langle \xi(t)\xi(t')\rangle = 2D\delta(t-t')$. The friction coefficient $\eta(x) = \eta_0[1 - \lambda \sin(2\pi x + \phi)]$, $|\lambda| < 1$, and ϕ determines the relative phase shift between friction coefficient and potential. The Fokker-Planck equation corresponding to Eq. (1) is given by [14]

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial J(x,t)}{\partial x} = \frac{\partial}{\partial x}\frac{1}{\eta(x)}\left(k_B T \frac{\partial}{\partial x} + [V'(x) - F(t)]\right) P(x,t), \quad (2)$$

where $J(x,t)$ and $P(x,t)$ are the current density, and probability density, respectively. The mean current

$$J = \lim_{t\to\infty} \frac{1}{\tau}\int_t^{t+\tau} dt \int_0^1 J(x,t)dx, \quad (3)$$

which is obtained numerically by solving Eq. (2) by the method of finite difference. The work done against the load, is given by $W = LJ$. The average input energy E is given by $E = \lim_{t\to\infty}(1/\tau)\int_t^{t+\tau} dt \int_0^1 F(t)J(x,t)dx$. The efficiency of the system in transforming the external force to useful work (storing potential energy) is [4]

$$\eta = \frac{W}{E} = \frac{LJ}{\lim_{t\to\infty}(1/\tau)\int_t^{t+\tau} dt \int_0^1 F(t)J(x,t)dx}, \quad (4)$$

where J is calculated from Eq. (3).

We now discuss the effect of finite frequency drive, spatial asymmetry, and inhomogeneous friction coefficient on the efficiency of energy transduction. It is observed that spatial asymmetry or a space dependent friction coefficient alone cannot enhance the nonadiabatic efficiency as compared to the adiabatic one in a rocked thermal ratchet. However, the interplay of both can enhance the efficiency in the nonadiabatic regime.

First we discuss the nature of the efficiency in an asymmetric ratchet in the absence of spatial frictional inhomogeneity ($\lambda = 0, \mu = 1$). This ratchet in contact with a thermal bath produces directed motion when rocked by a finite force. The direction of the current is dependent on both the direction of asymmetry as well as the frequency of the driving force [15]. When rocked adiabatically, the current shows a

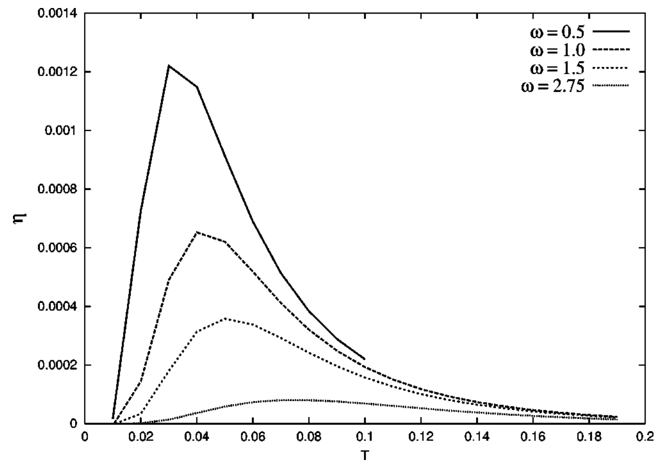

FIG. 1. Efficiency η vs temperature T for $A = 0.5$, $\mu = 1.0$, $\lambda = 0$, $L = 0.001$, and various values of ω. The curve for $\omega = 0.5$ follows the same trend as the other curves beyond $T = 0.1$.

maximum at some nonzero value of temperature. Even though the current shows a maximum the efficiency monotonically decreases with increasing temperature [4,7]. The situation changes in the nonadiabatic regime as shown in Fig. 1. Throughout this work temperature and frequency have been scaled appropriately to make them dimensionless [14]. In Fig. 1 we have plotted η vs T for various values of ω. It can be seen that, unlike the adiabatic case, η shows a maximum at a nonzero value of temperature. The value at which η peaks decreases with decreasing frequency, as it should. We have observed that, even though the efficiency peaks at a nonzero value of temperature in the nonadiabatic regime, the efficiency in the adiabatic regime (at $T = 0$) is much larger than the peak nonadiabatic efficiency.

In Fig. 2 we have plotted the efficiency as a function of the rocking frequency for various values of T (and A, in the

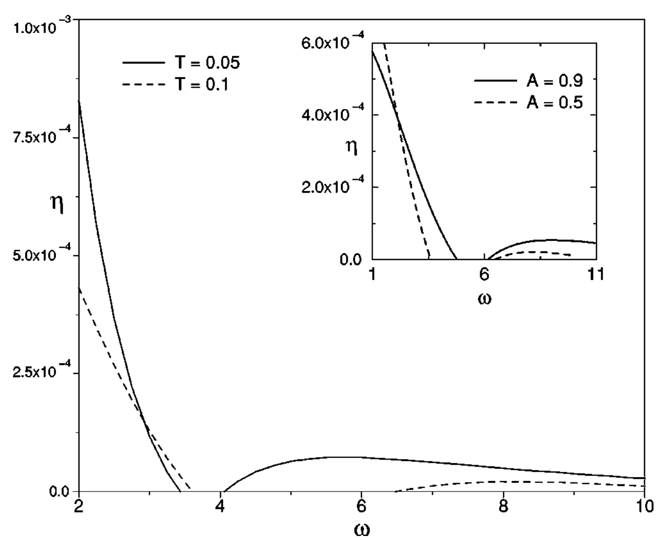

FIG. 2. Efficiency vs ω for two values of T at $A = 0.5$, $\mu = 1.0$, $\lambda = 0.0$, and $|L| = 0.005$. The inset shows variation of η with ω for two values of A at $T = 0.1$, all other parameter values remaining the same.

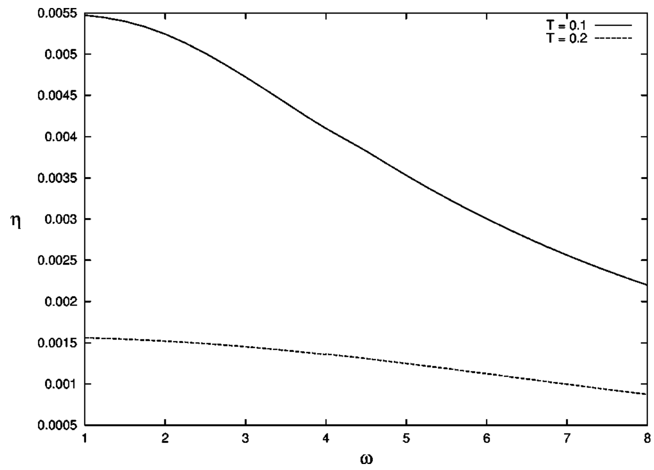

FIG. 3. Efficiency vs ω for $\mu=0$, $\lambda=0.9$, $\phi=0.6\pi$, $L=-0.012$, and for two values of T.

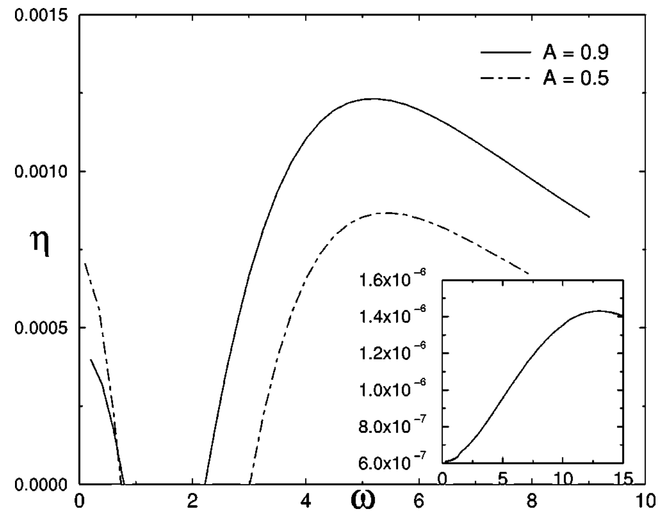

FIG. 4. Efficiency vs ω for two values of A. For other parameter values see text. The inset shows the variation of η vs ω for $A=1.5$, $T=0.4$.

inset). Current reversal as a function of frequency is a common phenomenon in a driven asymmetric ratchet. Since beyond a critical frequency the current reverses its direction [15], the load has been applied in the opposite direction in that regime so that work is done against the load. As shown in Fig. 2, for low frequencies, the efficiency shows a monotonic decrease with increasing frequency, the rate of decrease of η with ω being critically dependent on temperature T and amplitude A. In the current reversed regime, the efficiency shows a maximum with increasing ω, although its value is much less than the adiabatic efficiency. We have verified this fact by exhaustive numerical work with our given potential.

We now consider a system in which friction is space dependent with a *symmetric* potential profile. A unidirectional current results whenever $\phi\neq 0,\pi$ or 2π as discussed in Ref. [7]. In these models inversion symmetry is broken dynamically by space dependent friction. This system does not exhibit current reversal with any of the variables like T, A, or ω (in the absence of L) and hence we keep the load fixed in one direction for comparison of efficiency. In Fig. 3 we have plotted η vs ω for $A=0.5,\phi=0.6\pi$, and $L=-0.012$. For all values of T, λ, and ϕ, the efficiency monotonically decreases with increasing ω, i.e., for a given T the adiabatic efficiency is always maximum. In the two cases considered above ($\lambda=0$ with asymmetric potential and $\lambda\neq 0$ with symmetric potential) the adiabatic current is always more than the absolute value of the peak current in the nonadiabatic regime. The efficiency in our present case is mainly determined by the nature of the current and hence the result follows.

We now concentrate on frictional ratchets with an asymmetric potential profile $(\lambda\neq 0,\mu\neq 0)$. The efficiency characteristics of these ratchets have many counterintuitive features. In Fig. 4 we plot the efficiency as a function of ω for two values of the forcing amplitude A with $T=0.08$, $\lambda=0.9$, $\phi=0.2\pi$, and $L=0.015$. It can be clearly seen that the nonadiabatic efficiency is higher than the adiabatic efficiency, which is contrary to common belief that a rocked Brownian ratchet is inefficient in the nonadiabatic domain. This enhanced efficiency basically results from the *increase in the current with increase of the frequency in the current reversed regime*. The increase of current in this regime can be ascribed to the mutual interplay of spatial asymmetry and the space dependent friction coefficient [7]. The phase difference ϕ is chosen in such a manner that the steeper side has a lower friction coefficient than the slanted side. The inset shows a different qualitative behavior of η with increase of ω. Here $A=1.5$, $T=0.4$, $\lambda=0.1$, $\phi=0.2\pi$, and $L=-0.001$. In this parameter regime there is no current reversal. Here, as we increase the frequency from the adiabatic regime, η increases until it exhibits a maximum at very high frequency and decreases on further increasing the frequency. At high T and in the adiabatic regime, particles get sufficient kicks and enough time to cross the barrier on both sides, but the frictional drag on the steeper side is less. Hence the current flow is in the negative direction. On increasing the frequency, the Brownian particles get less time to cross the right barrier as they have to travel a larger distance to reach the basin of attraction of the next well than from the left side. Hence the net current increases. From the above argument it can easily be seen that the efficiency increases with increasing asymmetry (increasing μ) and vice versa, which we have checked in our work. For too high frequencies the particles do not get sufficient time to cross either of the barriers and the current decreases, which is reflected in the decrease of efficiency as shown in Fig. 4. Hence efficiency optimization at high frequency occurs not only in the current reversed regime but otherwise also. On decreasing the temperature the asymmetric ratchet effect becomes more pronounced and efficiency increases along with a shift of the peak efficiency to the lower frequency regime.

As in other previous cases, depending on the parameter values the efficiency (as a function of T) may or may not be maximized at finite temperature [4,7]. This optimum value (if a maximum exists) increases initially with increasing frequency and then for too high frequencies it decreases as discussed earlier. The temperature at which η peaks increases with increasing frequency as shown in Fig. 5. This shows

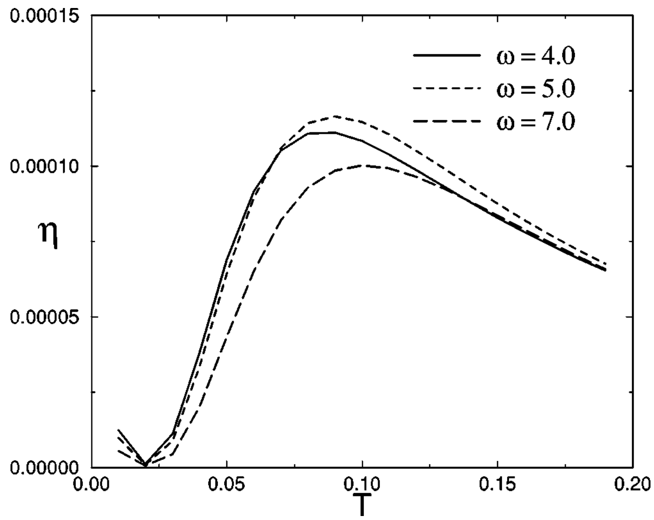

FIG. 5. Efficiency vs temperature for different values of ω. Here $\mu=1$, $\lambda=0.9$, $A=0.5$, $\phi=0.2\pi$, and $L=-0.001$.

that in contrast to the conventional wisdom where we associate high driving frequency and temperature with inefficient energy conversion, here both *high frequency* (nonadiabatic regime) and *high temperature enhance efficiency*.

In conclusion we have studied the efficiency of energy transduction in a forced frictional ratchet as a function of rocking frequency. Both nonmonotonic and monotonic behavior have been observed. In particular, the magnitude of the efficiency in the finite frequency regime may be more than the efficiency in the adiabatic regime. This implies that in these rocked ratchet systems quasistatic operation may not be efficient for conversion of input energy into mechanical work. Observation of a peak in the efficiency as a function of system parameters can be qualitatively attributed to the peak in the current and not to the behavior of the input energy, although the occurrence of a peak in current may not guarantee a peak in efficiency as observed earlier [4,7]. Here we have taken a simple ratchet type potential with space dependent friction coefficient to illustrate the above results. We do not rule out the fact that similar results can also be obtained in homogeneous systems for different ratchet potentials provided they exhibit larger absolute peak currents in the nonadiabatic regime. It is interesting to explore this possibility. It should be noted that in flashing ratchets, unlike rocking ratchets, the efficiency can show peaking behavior as a function of frequency. This is because in both zero frequency and the high frequency limits a flashing ratchet does not exhibit current [2]. A detailed study of input energy, work done, and dependence of the efficiency on other system parameters will be reported in the future.

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